Faculty of Mathematics and Information Science
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# Scalable Approaches to Bi-level and Dynamic Optimization Problems

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# Main Research Interests

- Games – all aspects
  - Human-like Game Playing (problem solving)
  - Multigame Playing
- Dynamic Optimiztion Problems
- Bi-level Optimizaton Problems

# **Human-Like** Problem Solving & **Human-Machine** Collaborative Problem Solving

- **Human-Like** Problem Solving
  - Intuition-based approaches (Knowledge patterns)
  - Focusing on goals /plans rather than particular actions
  - Multitasking
  - Highly selective search (efficient action preselection)
  - Search-Free or Shallow-Search based methods
  - Methods that rely on knowledge transfer (Transfer Learning)


- **Human-Machine** cooperation in problem solving
  - On-line steering of the algorithm
  - Human-Machine loop (trust building)
- **Human-Machine** co-learning
  - Human learning from collaborative problem solving

# Human-Machine Collaborative Problem Solving

- MCTS/UCT – simulation based approach
- Human player cooperates with UCT and has a budget of simulation batches

•Before each batch they can interfere with the algorithm:

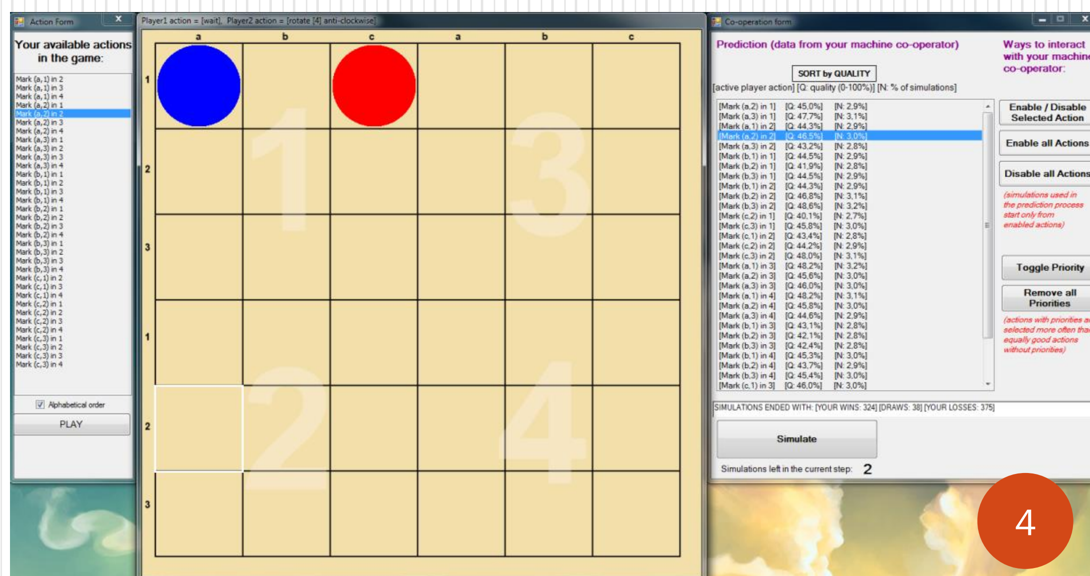
•**Enable/Disable actions** which simulations can start from (*narrowing the search*)

•**Toggling priorities of actions** making them simulated more often than the UCT formula says

•Humans **observe statistics** and make choices based on these statistics. In particular, they decide about the **move** to be played.

- **Human learns from collaborative problem solving**

- Creating a hybrid super-player
- Solving problems more effectively
- Improving the MCTS/UCT algorithm from the human input



# Security Games (SG)
# Stackelberg Equilibrium (Leader/Follower)

- A type of Pursuer-Evader game in a certain environment (usually represented as a graph)
- Two players: **Defender /Leader** (secures targets) and **Attacker /Follower** (attacks them)
- There are some targets to be defended/captured → **payoffs**
- **Imperfect information** (players reveal partial information about their strategies/behavior)
- The goal is to find **optimal Defender's strategy**
- **Non-zero-sum** game
- Both sides play a **mixed strategy**
- **BI-LEVEL: Follower knows Leader's strategy**
- **Follower is fully rational** and plays strategy that optimizes his reward
- **Follower breaks ties in his strategies in favor of leader's reward**

- Can be computed by solving continuous-discrete problem: **MILP**
- **Problems with scalability** of MILP solution

$for\ i \in \{D, A\}$

$\Sigma_i$ - a set of all pure strategies

$\Pi_i$ - a set of all mixed strategies, i.e. probability distribution over $\Sigma_i$

$$SE = (\pi_D, \pi_A) \in \Pi_D x \Pi_A$$

$$BR(\pi_D) = \arg\max_{\pi_A \in \Pi_A} U^A(\pi_D, \pi_A)$$

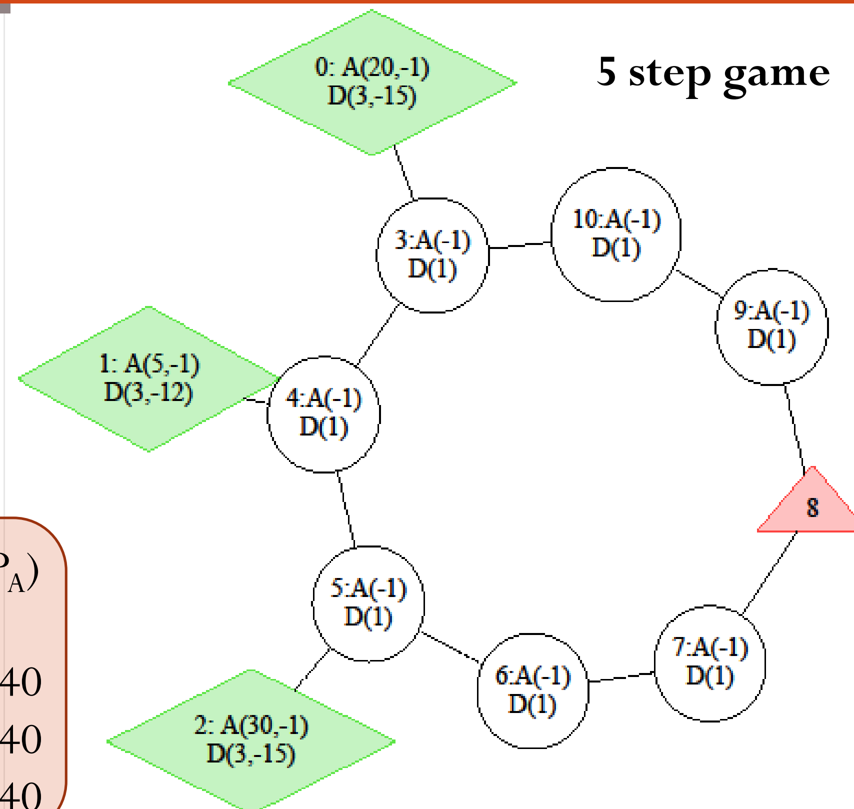
$$\arg\max_{\pi_D \in \Pi_D} U^D(\pi_D, BR(\pi_D))$$

# Security Games (SG)
# Stackelberg Equilibrium

Starting points: 0 & 8; Assets: 0, 1, 2

Rewards: non-targets and targets

Attacker can threaten either 0&1 or 1&2 ➔ Defender must follow *mixed strategy*




GAME EQUILIBRIUM:

| Prob. | Sequence | $E(P_D)$ |
|---|---|---|
| 0.4 | 0, 0, 0, 0, 0 | -15.00 |
| 0.6 | 3, 4, 5, 2, 2 | 3.00 |

| Prob. | Sequence | $E(P_A)$ |
|---|---|---|
| 1/3 | 7, 7, 6, 5, 2 | 11.40 |
| 1/3 | 7, 6, 6, 5, 2 | 11.40 |
| 1/3 | 8, 7, 6, 5, 2 | 11.40 |

**-4.20**

**-7.98**

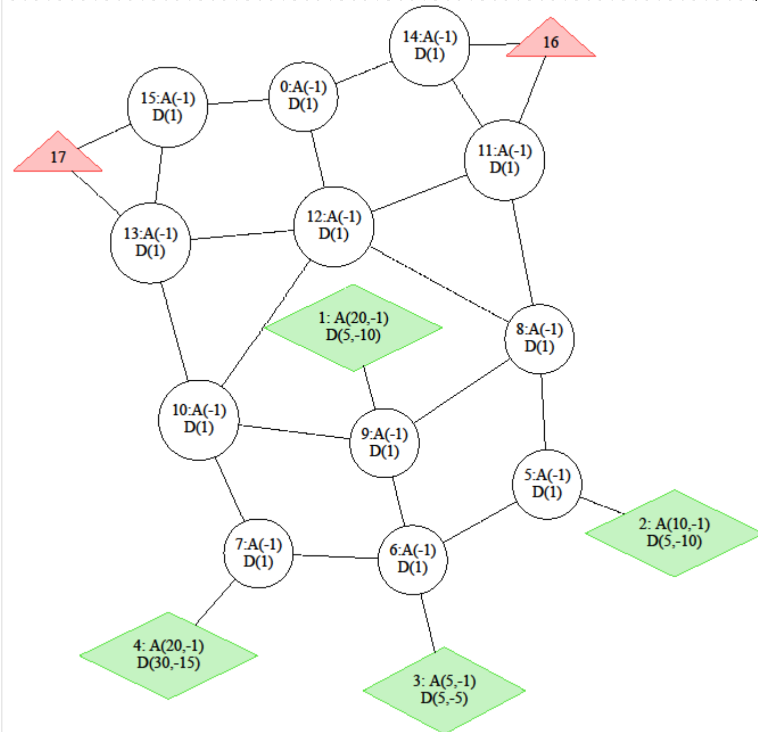
| Prob. | Sequence | $E(P_D)$ |
|---|---|---|
| 0.39 | 0, 0, 0, 0, 0 | -15.00 |
| 0.61 | 3, 4, 5, 2, 2 | 3.00 |

| Prob. | Sequence | $E(P_A)$ |
|---|---|---|
| 1/3 | 9, 9, 10, 3, 0 | 11.81 |
| 1/3 | 9, 10, 10, 3, 0 | 11.81 |
| 1/3 | 8, 9, 10, 3, 0 | 11.81 |

# Mixed-UCT method
# (Mixed strategy algorithm involving UCT)

**Idea**: iterative strategy improvement → equilibrium point

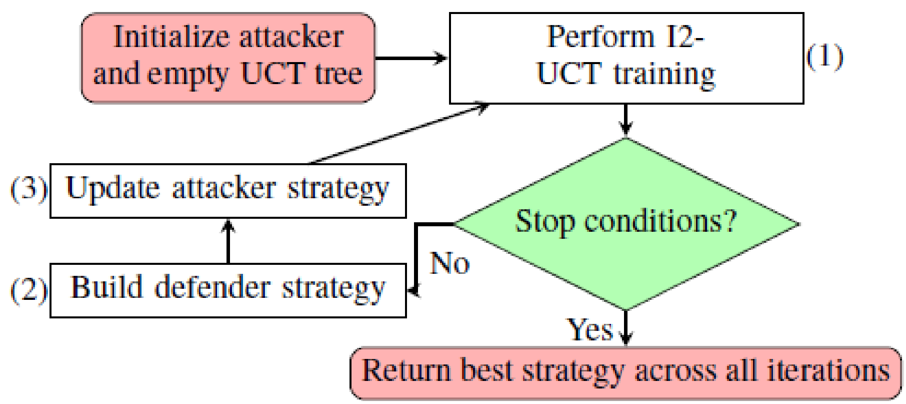
- 1. For the current Defender strategy calculate optimal Attacker strategy
- 2. **Perform UCT training against the currently calculated Attacker**
- 3. *** Store partial knowledge *** about current UCT solution
- 4. Calculate new Defender strategy based on collected knowledge
- 5. If not STOP then GOTO 1

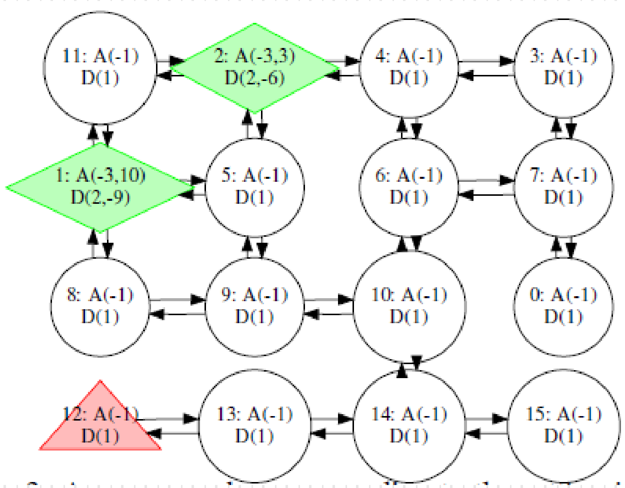


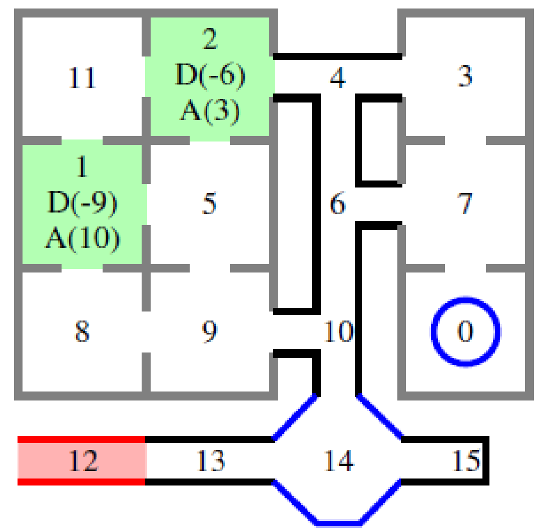

J. Karwowski, J. Mańdziuk, (2016), *Mixed strategy extraction from UCT tree in Security Games*, **22nd European Conference on Artificial Intelligence (ECAI'2016)**, The Hague, The Netherlands, 1746-1747

J. Karwowski, J. Mańdziuk, (2019), *A Monte Carlo Tree Search approach to finding efficient patrolling schemes on graphs*, **European Journal of Operational Research**, vol. 277, 255-268, Elsevier

# O2UCT - Improved UCT-based approach




**J. Karwowski, J. Mańdziuk, (2019),** *Stackelberg Equilibrium Approximation in General-Sum Extensive-Form Games with Double-Oracle Sampling*, ***International Conference on Autonomous Agents and Multiagent Systems (AAMAS-19)***, 2045-2047

# UCB1 (k-Armed Bandit)

- UCB = Upper Confidence Bounds (UCT = UCB applied to Trees)
- k-Armed Bandit Problem or k (one-arm) Bandits Problem
- Distributions of pay-offs $\{X_{j,t}\}_{t=1,2,\ldots}$ **$j=1,2,\ldots,k$** are fixed, but unknown
- How to maximize the total (long-term) reward?
- Exploration vs. exploitation balance

- First, play each arm once
- Then, use the following rule:

$$A^* = \arg\max_{i=1,\ldots,k} \left\{ \overline{X}_i + C\sqrt{\frac{\ln n}{n_i}} \right\},\ \ C = \sqrt{2}$$

# UCT – multiple simulations

Perform multiple simulations according to the following pattern:

Choose an action not yet selected (if exists)

If all had already been tried select action $a^*$

$$a^* = argmax_{a \in A(s)} \left\{ Q(s,a) + C\sqrt{\frac{\ln N(s)}{N(s,a)}} \right\}$$

*Q(s,a)* – the average result for a pair (*state*, *action*)

*N(s)* – the number of visits in state *s*

*N(s,a)* – the number of times action *a* was selected in state *s*

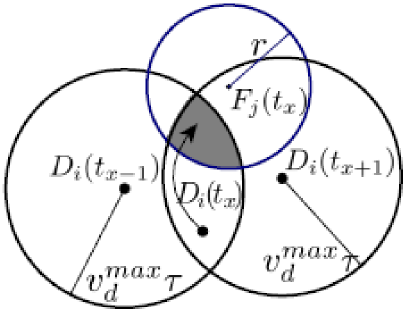
**Once the simulation is completed propagate the result back in the tree**

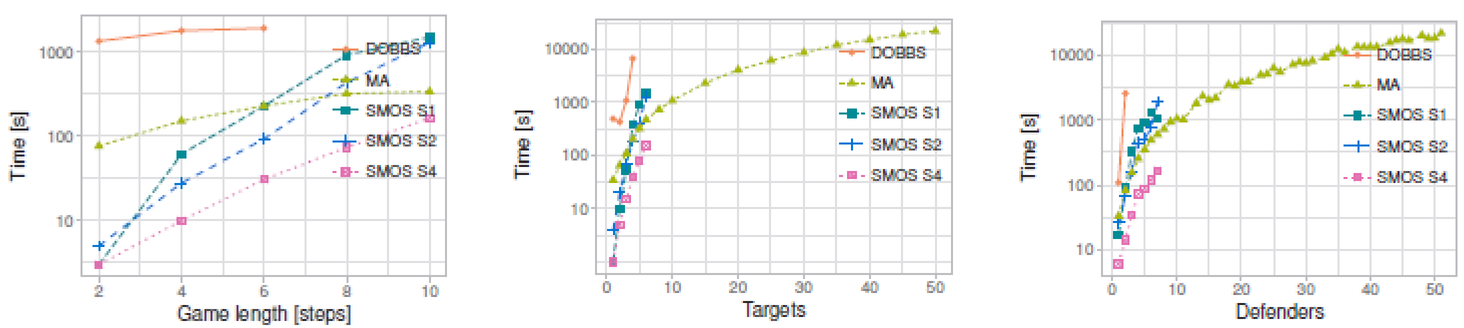
The saliency of C parameter

# SG with Moving Targets Evolutionary Approach

- $m \in \{2, 4, 6, 8, 10\}$
- $n_f \in \{1, 2, 3, 4, 5, 6, 8, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$
- $n_d \in \{max(1, n_f - 2), \ldots, n_f + 1\}$

- Real life transportation situations.
- Tourist harbour (ferry boats) in the Mediterranean Sea
- Number of ferries (targets) nf, patrolling boats nd
- Fixed ferry schedules on straingt routes
- Each chromosome encodes Defender's mixed strategy



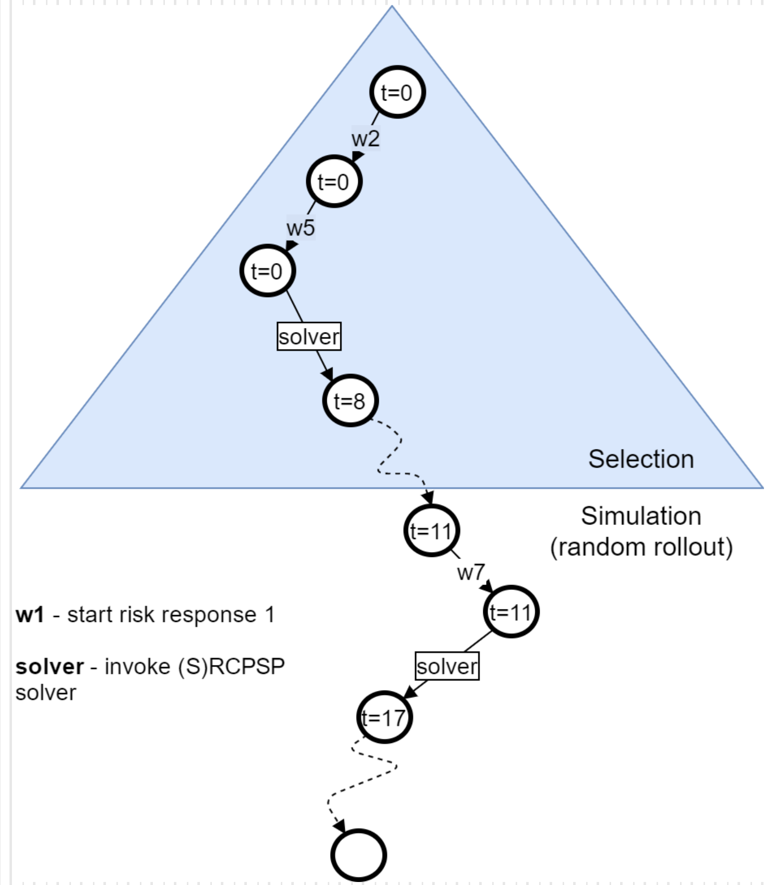

**J. Karwowski, J. Mańdziuk, A. Żychowski, F. Grajek, B. An (2019),** *A Memetic Approach for Sequential Security Games on a Plane with Moving Targets,* ***The Thirty-Third AAAI Conference on Artificial Intelligence (AAAI-19)***, 970-977

# Dynamic Optimization Problems

- Problems whose structure and / or parameterization change in time

- Risk-Aware Project Scheduling
- Dynamic Transportation Problems

- Monte Carlo Tree Search / UCT
- Evolutionary Algorithms
- Memetic Algorithms




**K. Walędzik, J. Mańdziuk, (2018)**, *Applying Hybrid Monte Carlo Tree Search Methods to Risk-Aware Project Scheduling Problem*, **Information Sciences**, vol. 460-461, 450-468, Elsevier

# Vehicle Routing Problem with Dynamic Requests Memetic Algorithm (EA + local optimization)

- *n* customers $v_1, v_2, \ldots, v_n$
- for each customer: demand, unload time (time required to unload cargo at customer's $v_i$), **arrival time $t_{vi}$** (time of arrival of the order from customer $v_i$)
- defined distance between each pair of customers
- fleet of m homogenous vehicles, each with identical capacity c
- speed of each vehicle is defined as one distance unit per one time unit
- depot with opening time $t_o$ and closing time $t_c$ ($0 <= t_o < t_c$)

**Goal:**

minimize the total routes' length of all vehicles according to the following constraints:

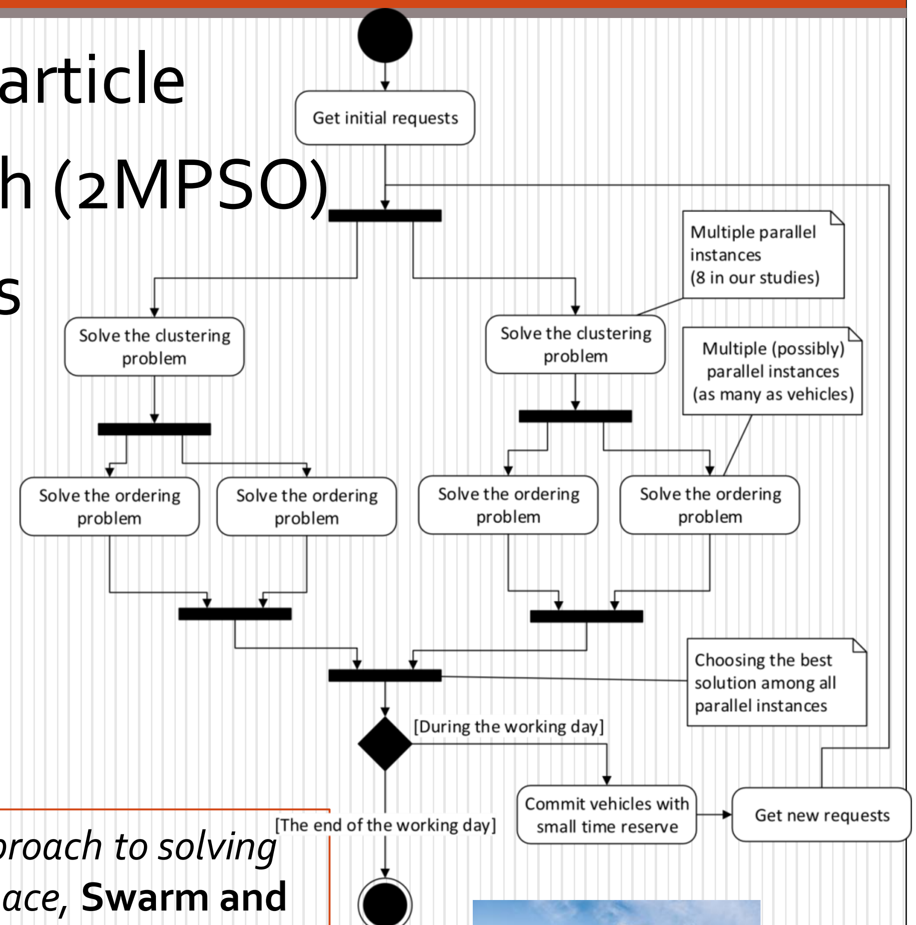
- each vehicle has to start from a depot after time $t_o$ and end its route in a depot before time $t_c$
- every customer has to be served exactly once and by one vehicle
- time of a vehicle arrival to customer $v_i$ has to be greater than $t_{vi}$ for all $i$
- the sum of customers' demands assigned to each vehicle must not exceed vehicle's capacity $c$



**J. Mańdziuk, A. Żychowski, (2016)**, *A Memetic Approach to Vehicle Routing Problem with Dynamic Requests*, **Applied Soft Computing**, vol. 48, 522-534, Elsevier.

# Vehicle Routing Problem with Dynamic Requests
# Particle Swarm Optimization with continuous coding

- Two-phase Multiple-Swarm Particle
- Swarm Optimization approach (2MPSO)
- Phase 1: clustering of requests
- Phase 2: routes optimization
- Continuous PSO coding
- Christofides / Fisher / Taillard dynamical benchmarks





**M. Okulewicz, J. Mańdziuk, (2019)**, *A metaheuristic approach to solving Dynamic Vehicle Routing Problem in continuous search space*, **Swarm and Evolutionary Computation**, vol. 48, 44-61, Elsevier

**M. Okulewicz, J. Mańdziuk, (2017)**, *The impact of particular components of the PSO-based algorithm solving the Dynamic Vehicle Routing Problem*, **Applied Soft Computing**, vol. 58, 586-604, Elsevier.


# Dynamic Vehicle Routing Problem with Traffic Jams

**Traffic is characterized by 3 distributions:**

Probability of encounter **p ∈ {0.02, 0.05, 0.15}** //for each edge

Length in steps **TTL := uniform<int>(2,5)**

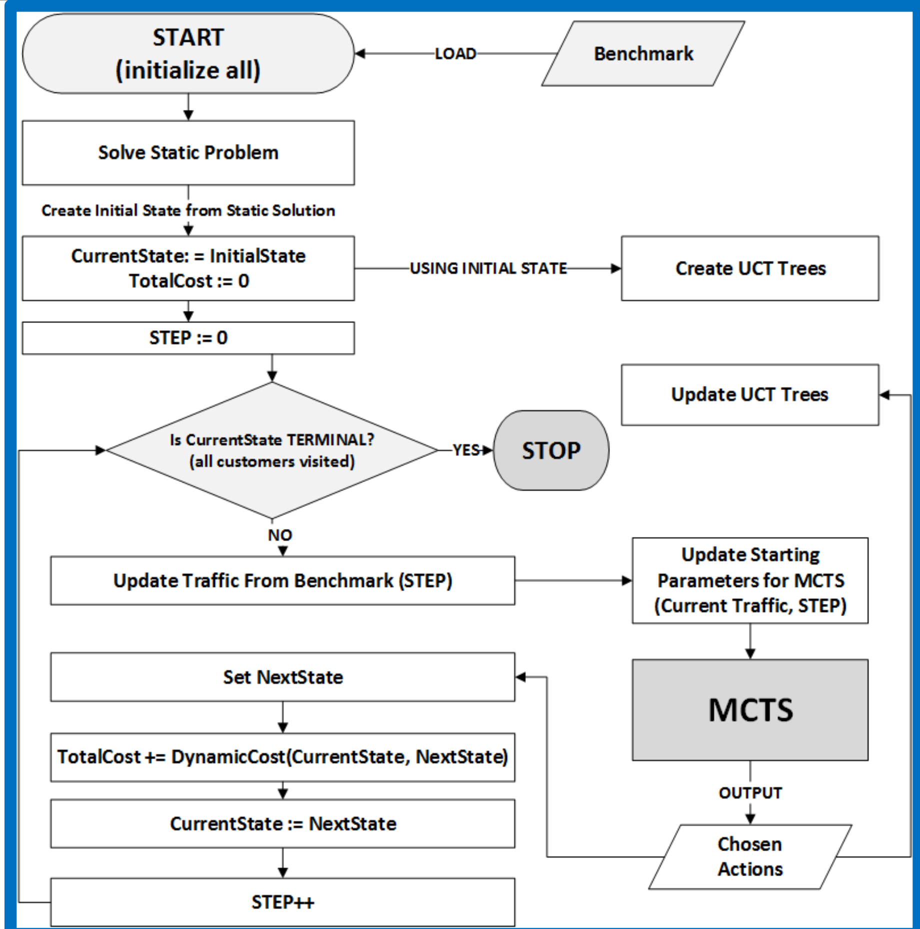
Intensity **I := uniform<int>(10,20)**

➔**The cost of an edge increases**

**Application of MCTS/UCT – UCT forest**

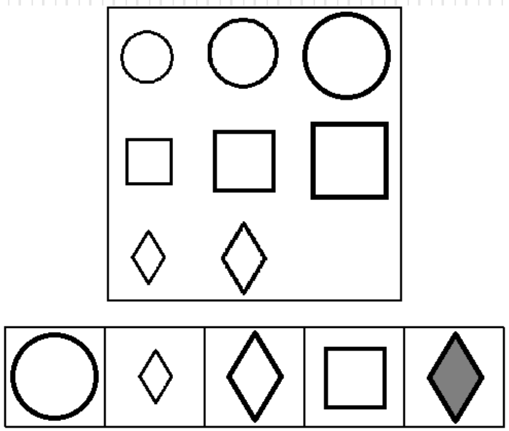
**One UCT tree per vehicle / Synchronization**

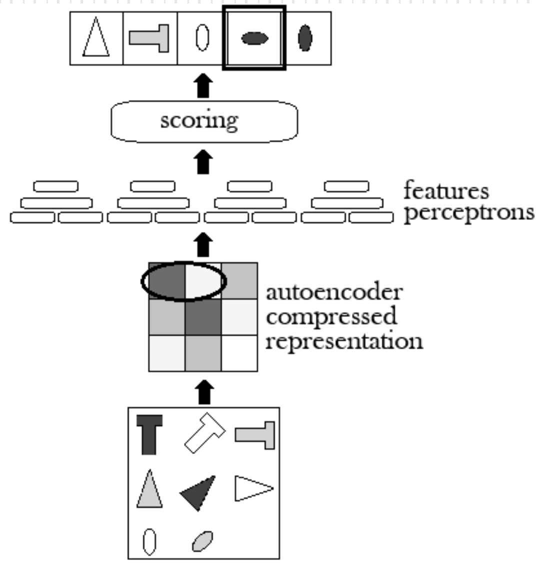
**UCT Actions:** local route modifications due to TJ: (swapping the two next clients in a route, move client from a jammed route to unjammed one, reverse the order of all clients in a route, etc.)

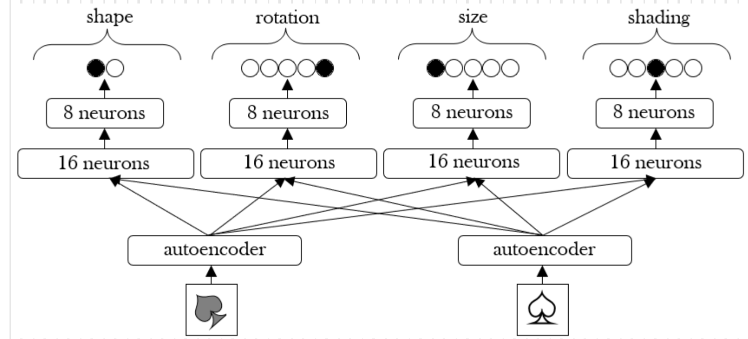


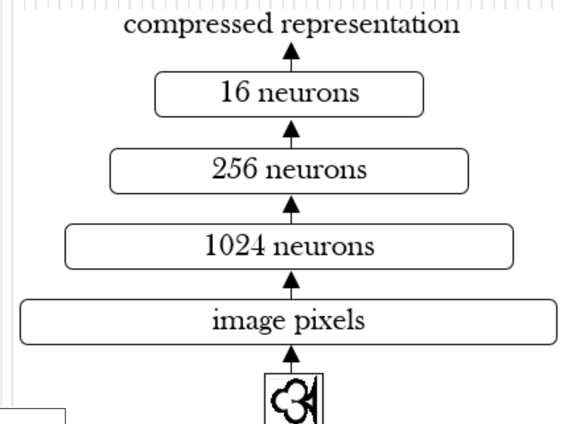

**J. Mańdziuk, M.Świechowski, (2017)**, *UCT in Capacitated Vehicle Routing Problem with Traffic Jams*, **Information Sciences**, vol. 406-407, 42-56, Elsevier

# Deep Autoencoders + MLPs ➔ Transfer Learning #1

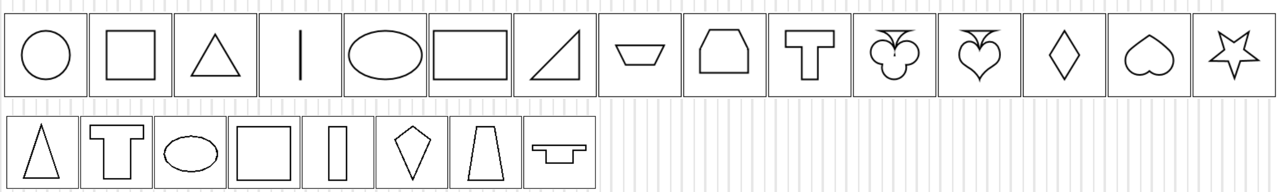





1. Different types of figures used in training and testing
2. Different types of IQ problems in training and testing



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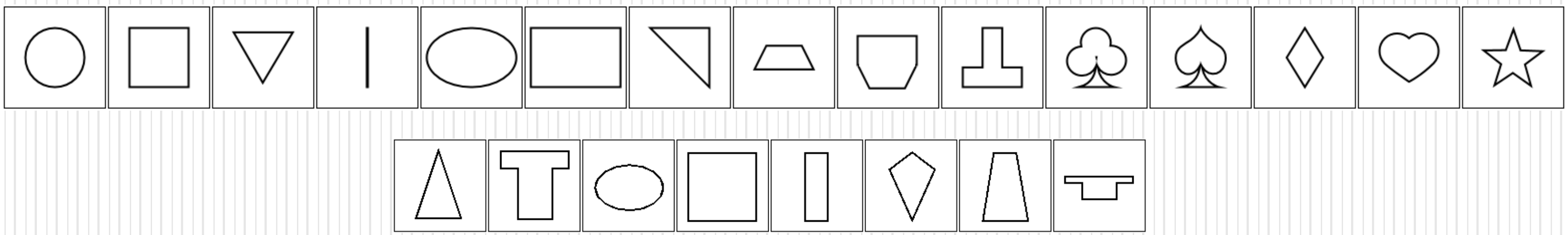
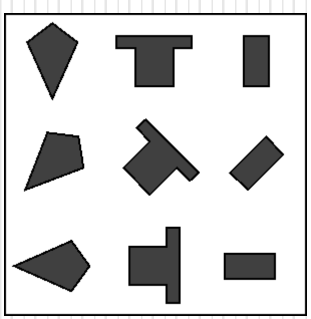
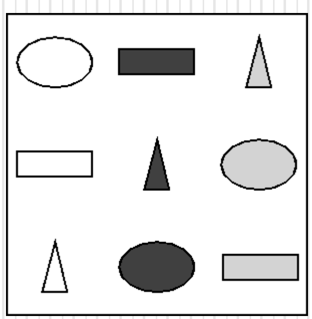
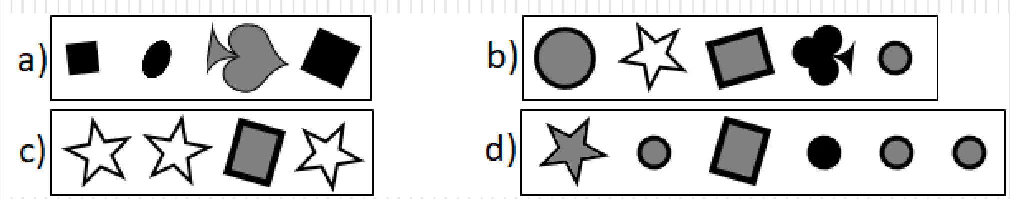
# Deep Autoencoders + MLPs ➔ Transfer Learning #2

1. Different types of figures used in training and testing
2. Different types of IQ problems in training and testing










No additional training or tuning of the system

# My Research Team

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# SUMMARY

- **GAMES / MULTISTEP DECISION MAKING PROBLEMS**
  - Human-like problem solving
  - Multigame Playing
- **BI-LEVEL OPTIMIZATION (Scalable solutions)**
  - MCTS/UCT simulation approaches
- **DYNAMIC OPTIMIZATION PROBLEMS (Scalable solutions)**
  - MCTS/UCT simulation approaches
  - Evolutionary Algorithms
  - Memetic Algoritms

**Thank You!**

**Questions?**